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# Introduction

LCH.Clearnet (hereafter LCH) proposes to begin clearing inflation based derivatives. In particular, the proposed products are Zero Coupon Inflation Indexed Swaps (ZCIIS). These are very standard in the market and are based on specific price indices. LCH proposes to offer the product in GBP (RPI index), EUR (Eurowide HICPxT and French CPIxT) and USD (CPI), which represent the major liquid markets in inflation swaps.

LCH proposes to support non-standard lags out to a maximum of 1 year lag. Non standard lags only represent a small proportion of the trade flows in the market, especially where the counterparties are restricted to mandatory clearing counterparties.

The products are long dated and LCH proposes to clear tenors out to 30 years, with 50 years in Sterling. The market is quite different in structure to IRS and Swaptions, as “real money” is a huge player and tends to build up very structural positions which are held to maturity, as they form hedges against their real business.

# Pricing Zero Coupon Inflation Swaps

The valuation of these products is relatively simple as there are only two market inputs, the predicted future level of the index and the discount rate. Discounting in a cleared world would be at standard OIS rates, the only complexity here being the length of the trades may exceed the maximum quoted length of the OIS market.

The future prediction of the price index can be estimated from bootstrapping market instruments. ZCIIS are widely traded for standard tenors and market data is readily available, including historic time series. In addition inflation linked government bonds are widely traded and could also be used as a source of data though the bonds are more complex to use with bootstrapping and in some cases include embedded optionality that lies outside the capabilities of a simple model. Section **Error! Reference source not found.** of this document details the calculation of the forward price index.

One complexity in the forecast price curve is that seasonality must be taken into account. This is requires as there are expected cyclical cost shifts in some components of the index. A simple example is home heating costs are expected to rise in winter and fall in summer. Seasonality can be estimated using econometric methods on historic indices, or taken as market data inputs direct from members which is the proposed method. The impact of seasonality on pricing is most noticeable at the short end of the curve, and as the product length rises it more and more averages itself out.

Where the fixing lag for the final price index follows the market standard, the above are sufficient to exactly price the product. The PV of a trade is therefore given by discounting the simple payout back to the valuation date:

Where

: Index level observed at the swap fixing date

: Reference Index level observed at the fixing date = Swap start date – Lag

Pay or receive flag related to the Inflation leg:

: Fixed rate that nullifies the ZCIIS PV at inception

: Swap length in integer years

: Swap notional amount

: Discount factor from swap payment date to pricing date

Non-standard lags introduce small convexity adjustments that theoretically need to be introduced to amend the projected price index. The reasoning is the same as for the more well understood LIBOR in arrears and CMS adjustments in pure rates. The convexity adjustment in ZCIIS with non-standard lags is dependent on the volatility of rates, the volatility of the price index, the correlation between rates and price index, the length of the deal and the length of the lag. In order to correctly evaluate the adjustment a more complex model is required, such as Jarrow-Yildrim, but these are difficult to calibrate and slow to converge. One proposal suggested is to use member supplied curves for different lags that incorporate the adjustments implicitly. Research papers in the public space however show that the size of the adjustment is only significant for lags much longer than 1 year which is the proposed maximum length. Any convexity adjustment would cancel itself out in a VAR calculation. It is therefore proposed that the convexity adjustment can be ignored for LCH scope.

# Inflation Curve

The forward CPI level of the Inflation index is determined from an inflation curve. In general the Inflation curve should change depending on the Inflation Index lag. It has been confirmed that for lags up to one year, using one curve for all lags instead of one curve per lag does not add a large error. The maximum error for a 1Y lag is 6 basis points.

ZCIIS prices will be sourced from LCH members and processed to have one unique curve per Inflation index for all lags. LCH will source quotes from its members of ZCIIS Rates for different standard market maturities. These instruments implicitly contain the information on the Consumer Price Index (CPI) at forward dates.

## Tenors

### Variation Margin Curves

For Variation Margin (VM), curves will have a monthly granularity in the short end of the curve; up to 2 years where seasonality matters the most. Members will therefore provide monthly quotes that include seasonality.

|  |  |  |  |
| --- | --- | --- | --- |
| GBP | USD | EUR | FRF |
| 1M | 1M | - | 1M |
| 2M | 2M | 2M | 2M |
| 3M | 3M | 3M | 3M |
| 4M | 4M | 4M | 4M |
| 5M | 5M | 5M | 5M |
| 6M | 6M | 6M | 6M |
| 7M | 7M | 7M | 7M |
| 8M | 8M | 8M | 8M |
| 9M | 9M | 9M | 9M |
| 10M | 10M | 10M | 10M |
| 11M | 11M | 11M | 11M |
| 1Y | 1Y | 1Y | 1Y |
| 13M | 13M | 13M | 13M |
| 14M | 14M | 14M | 14M |
| 15M | 15M | 15M | 15M |
| 16M | 16M | 16M | 16M |
| 17M | 17M | 17M | 17M |
| 18M | 18M | 18M | 18M |
| 19M | 19M | 19M | 19M |
| 20M | 20M | 20M | 20M |
| 21M | 21M | 21M | 21M |
| 22M | 22M | 22M | 22M |
| 23M | 23M | 23M | 23M |
| 2Y | 2Y | 2Y | 2Y |
| 3Y | 3Y | 3Y | 3Y |
| 4Y | 4Y | 4Y | 4Y |
| 5Y | 5Y | 5Y | 5Y |
| 6Y | 6Y | 6Y | 6Y |
| 7Y | 7Y | 7Y | 7Y |
| 8Y | 8Y | 8Y | 8Y |
| 9Y | 9Y | 9Y | 9Y |
| 10Y | 10Y | 10Y | 10Y |
| 12Y | 12Y | 12Y | 12Y |
| 15Y | 15Y | 15Y | 15Y |
| 20Y | 20Y | 20Y | 20Y |
| 25Y | 25Y | 25Y | 25Y |
| 30Y | 30Y | 30Y | 30Y |
| 35Y | - | - | - |
| 40Y | - | - | - |
| 45Y | - | - | - |
| 50Y | - | - | - |

### Initial Margin curves

Initial Margin (IM) curves will be a subset of Variation Margin curves. Tenors will be the same as VM curves at the exception of the short end of the curve where only the 1Y and 2Y tenors remain.

|  |  |  |  |
| --- | --- | --- | --- |
| GBP | USD | EUR | FRF |
| 1Y | 1Y | 1Y | 1Y |
| 2Y | 2Y | 2Y | 2Y |
| 3Y | 3Y | 3Y | 3Y |
| 4Y | 4Y | 4Y | 4Y |
| 5Y | 5Y | 5Y | 5Y |
| 6Y | 6Y | 6Y | 6Y |
| 7Y | 7Y | 7Y | 7Y |
| 8Y | 8Y | 8Y | 8Y |
| 9Y | 9Y | 9Y | 9Y |
| 10Y | 10Y | 10Y | 10Y |
| 12Y | 12Y | 12Y | 12Y |
| 15Y | 15Y | 15Y | 15Y |
| 20Y | 20Y | 20Y | 20Y |
| 25Y | 25Y | 25Y | 25Y |
| 30Y | 30Y | 30Y | 30Y |
| 40Y | - | - | - |
| 50Y | - | - | - |

## Instrument definition

Instruments defined to build the Inflation curve will be configured as Zero Coupon Inflation Swap with the following date definitions:

* Swap Start date
  + Market data date + Currency lag
  + Following adjustment
* Swap End date
  + Swap Start date + Tenor
  + Unadjusted
* Swap Fixing date
  + Swap End date – Standard lag
  + Unadjusted
* Swap payment date
  + Swap End date
  + Modified Following adjustment

Currency lag is defined as:

* 2 business days for EUR and USD currencies
* 0 business day for GBP currency

In simple terms the quotes should be for spot starting swaps.

## Compounding

For whole year quotes, the compounding index will be computed as the integer number of years in the swap. Where the period covered is an inexact number of years, the compounding index will be measured as (Number of Months/12).

As an example a 3 month rate of 2.6% would result in a fixed rate payment of (1+2.6%)^(0.25) – 1.

## Lag adjustment

The forward CPI level of the Inflation index is determined from an inflation curve. In general the Inflation curve should change depending on the Inflation Index lag. It has been confirmed that for lags up to one year, using one curve for all lags instead of one curve per lag does not add a large error.

One curve per index will be configured in the system and will use the standard market lag for each index:

* 2M for GBP (RPI index)
* 3M for EUR (Eurowide HICPxT and French CPIxT) and USD (CPI)

## Seasonality

Inflation is expressed at different fixed maturities but between two maturities the index does not evolve with a constant rate. Inflation is subject to seasonality and it is therefore necessary to adjust it on a month to month basis.

Inflation curves will be configured to use Multiplicative seasonality. Seasonality surfaces will be sourced from LCH members and processed to build one unique seasonality surface per index. Members should quote each of the 12 months.

The method used to aggregate seasonality surfaces is to keep or reject all monthly adjustments from one member. The decision to keep or reject a contributor will be based on how far away from the averaged surfaces a member is.\*

\*Replication of the cleansing algorithm in Appendix.

## Interpolation

When pricing an Inflation Swap, its fixing date will not always fall on a pillar of the Inflation curve; therefore an interpolation is needed to handle those cases.

The convention to interpolate CPIs between two dates is to do a Log-Linear interpolation on the CPI levels from adjacent pillars around the date to interpolate. Interpolation on the curve is always done for the 1st of a month. There are no distinctions at that point between piecewise and linear indices.

The following formula is used to interpolate a CPI when using multiplicative seasonality:

Where

: CPI of left pillar of the current bucket

: CPI of right pillar of the current bucket

: Seasonality adjustment for month p

: Seasonality factor for left pillar month of the bucket

: Seasonality factor for right pillar month of the current bucket

: Number of months from the fixing date to the previous pillar (fixing date is a month, convention is 1st of the month)

: Number of months between left and right curve pillars

For the linear case, after interpolating CPIs for current Fixing date month and the following month; a linear interpolation is done using days from the swap end date month to weight the two CPIs. The market convention is to round the interpolated index level to 5 decimals when the index has been published.

Where

: Swap End date

L: Payment lag

: Day of the Swap End date

: Number of days in the Swap End date month

## Stripping

CPI levels will be implied from ZCIIS quotes so that the zero coupon inflation swap NPV equals zero.

### CPIs

ZCIIS quotes can be used to construct the forward CPI level using the following formula:

Where:

: Maturity year of pillar n (calculation as defined in paragraph 3.3)

: Zero Coupon Inflation Rate for pillar n

: Reference CPI for a Swap (vary function of the Index lag: 2M for UK RPI, 3M for the others)

CPI level is expressed for a month in case of Piecewise indices but for a specific date in case of Linear indices.

### CPI Rates

CPI rates are close to market quotes in the sense that they are rates and can be used to calculate index levels. They do not incorporate seasonality effect in them and are built using the last published index as a reference CPI.

CPI rates are deduced from Index levels and are rebased using the last published index at that time by implying back the rate.

Assuming that CPIs for all pillars are known, Market practice is to use the Yield formula with 30/360 by month basis convention. The CPI rates are calculated using the following formula:

Or

Where

: CPI rate of left pillar of the current bucket

: CPI rate of right pillar of the current bucket

: Seasonality adjustment for month p

: Seasonality factor for left pillar month of the bucket

: Seasonality factor for right pillar month of the current bucket

: Number of months from the fixing date to the previous pillar (fixing date is a month, convention is 1st of the month)

: Number of months between left and right curve pillars

Time to maturity from left pillar of current bucket to last published index using 30/360 by month convention (Diff Years \* 12 + Diff Months) / 12

Time to maturity from right pillar of current bucket to last published index using 30/360 by month convention (Diff Years \* 12 + Diff Months) / 12

Time to maturity from month p to last published index using 30/360 by month convention

### Reference CPI

There are two different methods to calculate, it is even an end-of-month value or a value interpolated between two consecutive months.

For the end-of-month case, the reference price level is the published index value specified by the index lag. For example for index UKRPI with a standard 2 months lag on the 12th of December 2012, the reference price level is the fixing observed in October 2012.

For the interpolated case, a new value of the reference price level is calculated every day. For the first day of the month, the reference value will be the same as the end-of-month case. But for the following days, that reference value is calculated by interpolating between that index and the next one.

The reference price level is calculated following this formula:

Where

: Current month of the market data date

: Index lag in month

: Day of the current month

: Number of days in the current month

### Inflation Curve dates

Dates at which CPIs are expressed are generally the instrument fixing dates. In case of piecewise index such as UK RPI or EU HICPxT this is always the case. For linear indices such as FR or US CPI; the interpolation is done in two steps:

* interpolate the index level from the Inflation curve on the current and following month of the fixing date
* linearly interpolate the two CPIs to express it on the fixing date

In that case, CPIs are expressed on the following month of the fixing date by solving a system of equations:

* linear interpolation between current month and following month of the swap fixing date
* log linear growth between last pillar and current month
* log linear growth between last pillar and following month

As an example a 2Y Swap based on US CPI index that fixes on the 12th of April 2015 will have its CPI calibrated on May 2015.

The following equations will be used to calibrate the 2Y instrument:

Where:

: Last pillar of the curve, 1Y Instrument

: Current month of the swap fixing date, **unknown variable**

: Following month of the swap fixing date, **unknown variable**

: Trend, **unknown variable**

: Seasonality for April

: Seasonality for May

: Day count from April 15 to May 14

: Day count from May 15 to May 14

When a left pillar is not available like for a 1Y Swap with no monthly tenors defined on the curve, the last known fixing will be used.

Assuming a 1Y Swap based on US CPI index fixes on the 12th of April 2014 and the last known fixing is February.

The equations become the following to calibrate the 1Y instrument:

In case of monthly quotes used to build the curve, the stripping is easier and no system of equation needs to be solved to calibrate the curve. The current month is also the last calibrated point and a single equation can be used.

Assuming a 3M Swap based on US CPI index fixes on the 12th of April 2014 and the last calibrated instrument was a 2M Swap.

The equation becomes the following:

Known CPI from the calibrated 2M instrument

Therefore

# Sensitivities

Where

: CPI of trade, calculated from end date and using the deal lag

: Reference CPI of the deal, calculated from swap start date and using the deal lag

Fixed rate

Nominal

Time to maturity for the swap following using 30/360 by month basis convention

Sensitivities for Piecewise and Linear indices follow the same equations. In the case of Linear indices, is linearly interpolated from two CPIs and the chain rules need to be applied for each index. One other major difference between the two types of index is that Linear indices curve stripping create a dependency to previous pillars, therefore the translation of a CPI rate sensitivity into a Par sensitivity is done using a Jacobian/Hessian matrix. There is not a one to one relation between a CPI rate sensitivity and a Par sensitivity on a tenor (i.e. a CPI rate sensitivity for Linear indices on the 3Y tenor will create Par sensitivity on 1Y, 2Y and 3Y tenors)

## Inflation Sensitivities

### CPI Delta

CPI Delta is the derivative of the present value of a trade regarding the CPIs on the curve. Following the chain rule; it is a composition of two partial derivatives: the derivative of the PV against the CPI used by a trade and the projection on the two surrounding tenors of the curve.

Where

: CPI of the deal, calculated from swap start date and using the deal lag

: CPI for pillar n of the Inflation Curve

Calculation of CPI Delta is the same for piecewise of linear indices; the only difference is that in case of linear indices, the PV is sensitive to two CPIs.

### CPI Rate Delta

CPI Rate Delta is the derivative of the present value of a trade regarding the CPI rates on the curve. Following the chain rule; it is a composition of three partial derivatives: the derivative of the PV against the CPI used by a trade, the translation of that CPI sensitivity into a CPI Rate sensitivity and its projection on the two surrounding tenors of the curve.

Where

: CPI of the deal, calculated from swap start date and using the deal lag

: CPI Rate of the deal

: CPI Rate for pillar n of the Inflation Curve

### Par Delta

Par Delta is the first derivative with respect to Par rates. It can be calculated as a composition of four derivatives: the derivative of the PV against the CPI observed, then composed with the derivative of that CPI against its CPI rate then the projection on the CPI rates of the curve and subsequently taking the derivative of the CPI rate with respect to the par rate.

Where

: CPI of the deal, calculated from swap start date and using the deal lag

: CPI Rate of the deal

: CPI Rate for pillar n of the Inflation Curve

: Par Rate for pillar n of the Inflation Curve

In the case of linear indices, there is not a one to one relationship between CPI rate and Par rate. Multiple CPI rate sensitivities contribute to a Par rate sensitivity; therefore Par Delta can be written as a sum of contributions:

Assuming fixing(s) for a ZCIIS falls in between date 3 and 4, Par Deltas will look as follow:

|  |  |
| --- | --- |
| Piecewise index | Linear index |
|  |  |

In the linear index case, sensitivity on Par rates 1 and 2 is created from adjacent CPI rates 3 and 4 (adjacent tenors to the trade fixing).

### CPI Gamma

CPI Gamma is the second derivative of the PV against the CPI of the Inflation curve. Gamma can be represented as a matrix:

A trade can only be at maximum sensitive to two CPIs of the Inflation curve. The matrix is symmetric; the cross sensitivities are identical: .

Bug in Murex: CPI index Delta and Gamma are not projected on the Inflation curve, CPI index is a constant therefore Murex Gamma is always equal to zero.

Diagonal term:

In the case of a trade fixing on a curve pillar the projection on the curve doesn’t exist anymore and Gamma equals zero.

Cross term:

Details:

### CPI Rate Gamma

CPI Gamma is null in some cases, therefore a way to calculate the second derivative is to skip the first part of the chain rule and directly express the PV function of CPI rates. Gamma can be represented as a matrix:

A trade can only be at maximum sensitive to two CPIs of the Inflation curve. The matrix is symmetric; the cross sensitivities are identical: .

Diagonal term:

Or

Cross term:

or

Details:

### Par Gamma

Par Gamma can be calculated as an extension of CPI Rate gamma with the chain rule. There is a major difference in Piecewise and Linear indices in the way the matrix is filled:

Piecewise Gamma:

A trade can only be sensitive to two Par rates of the Inflation curve at maximum. The matrix is symmetric; cross sensitivities are identical: .

Diagonal term:

Cross term:

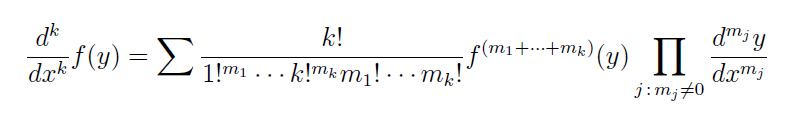
Details:

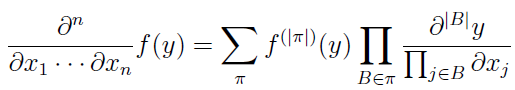
Linear Gamma:

A trade can only be sensitive to two CPIs of the Inflation curve but those will create Par sensitivity on all previous tenors. The matrix is symmetric; cross sensitivities are identical:

, and .

The chain rule for Gamma can be written as a sum of contributions; by using Francesco Faà Di Bruno’s formula who generalized the chain rule formula to higher derivatives, we can write:





And deduce:

Where:

Diagonal terms:

Cross terms:

or

Cross Gamma details:

******

## Interest Rate Sensitivities

Piecewise or linear interpolation types do not change the interest rate risk; the interest rate risk only comes from the discounting risk of a unique cash flow. It is therefore null when the PV is null and will increase based on the cash flow size.

### Zero Delta

In the general case where the payment date of a ZCIIS falls in between two tenors of the IR curve, the sensitivities related to the two tenors can be written as follow:

Where:

Date of right tenor

Date of left tenor

Swap payment date

Time to maturity from payment date to pricing date using ACT/365 basis convention

Present value of the deal

In the case where the payment date falls on a tenor of the IR curve, the sensitivity can be resumed to one unique equation:

### Zero Gamma

Gamma is a diagonal matrix where for a single trade only four terms are not null. In case the payment date falls on an IR tenor, the matrix is filled up with only one term (.

Diagonal term:

Cross term:

## Cross Gamma Sensitivities

Inflation and Interest rate risks are independent, therefore the order of derivation is not important and Cross Gamma can be treated as the Inflation Delta projected composed with the Interest Rate Delta.

Current assumptions are that LCH will use Cross Gamma between Par Inflation rates and Zero Coupon IRS rates. Cross Gamma =

LCH IRS curves always have an overnight point (O/N for VM or 1D for IM curves); therefore a trade payment date can’t be before the first pillar of the IRS curve. Based on eligibility rules, Inflation trades can’t go beyond 50Y and there is a tenor above any payment date of trade (no extrapolation is needed). There are therefore six combinations of date on which to calculate Cross Gamma.

### Par-ZC Cross Gamma

The Cross Gamma matrix is symmetric; deriving on the zero coupons IRS space and then Par Inflation rate is identical as doing it in the other sense.

The same formulas defined for Par Inflation Delta and Zero IRS Delta can be applied. The same differences are found between linear and piecewise indices: cross gamma is composed of four terms for piecewise vs two columns for linear indices.

|  |  |
| --- | --- |
| Piecewise index | Linear index |
|  |  |

### CPI-ZC Cross Gamma

# Inflation Sensitivity Formulas

Piecewise or Linear interpolation does not change the sensitivity formulas up to the Par sensitivities. In case of a linear interpolation, sensitivity formulas need to be applied twice with the correct linear coefficient for each one. Formulas that define interpolation and the link between CPI and CPI rates (Z) are identical for linear and piecewise interpolation.

## Piecewise Interpolation

In case of a piecewise interpolation, a trade only requests one index level. Choosing a log linear index level interpolation, in the general the PV formula can be written case as follows:

Or

Or

Where

: Reference CPI, calculated from spot date and using the standard lag

: Reference CPI of the deal, calculated from swap start date and using the deal lag

: Par Inflation rate for the left pillar

: Par Inflation rate for the right pillar

: Seasonality factor for left pillar month

: Seasonality factor for right pillar month

: Seasonality adjustment for month p

: Number of months from the fixing date to the left pillar

: Number of months between left and right pillars

Fixed rate

Nominal

Time to maturity for the left pillar using 30/360 by month basis convention

Time to maturity for the right pillar using 30/360 by month basis convention

Time to maturity for the swap following using 30/360 by month basis convention

Discount factor from the payment date to the pricing date using ACT/365 basis convention

Pay or receive flag related to the Inflation leg:

### CPI Delta

CPI Delta for piecewise indices is the composition of two derivatives: the derivative of the PV against the CPI observed and the projection on the CPIs of the curve.

1. Sensitivity on the Index level
2. Projection on Index levels of the curve

In between two pillars

Before first pillars

On a pillar

### CPI Rate Delta

CPI Rate Delta can be calculated as the composition of three derivatives: the derivative of the PV against the CPI observed, then composed with the derivative of that CPI against its CPI rate and finally the projection on the CPI rates of the curve tenors.

1. CPI against its CPI rate
2. CPI rate against curve pillars

In between two pillars:

On / before first pillar:

### Par Delta

Par Delta can be calculated as a composition of four derivatives: the derivative of the PV against the CPI observed, composed with the derivative of that CPI against its CPI rate, projected on the CPI rates of the curve and subsequently taking the derivative of the CPI rate with respect to the par rate.

The first three partial derivatives have been calculated in the previous steps. The forth derivative in the chain is calculated below.

**Not needed**

In between two pillars:

On a pillar:

Before first pillar:

### CPI Rate Delta (direct derivation)

In between two pillars:

The general formulas when an index level falls in between two curve pillars are the following:

Before first pillar:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

On a pillar:

When an index level falls on a pillar date, the trade is only sensitive on that unique pillar and the delta formula is much simpler.

### Par Delta (direct derivation)

In between two pillars:

The general formulas when an index level falls in between two curve pillars are the following:

Before first pillar:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

On a pillar:

When an index level falls on a pillar date, the trade is only sensitive on that unique pillar and the delta formula is much simpler.

### CPI Gamma

CPI Gamma is the second order derivative with respect to CPI levels. It is a symmetric matrix; where only four terms are not null.

Based on the chain rules define above, only two new terms have not been defined: and

1. Diagonal terms:

In between two pillars

Before first pillars

On a pillar

1. Cross term:

In between two pillars

Before first pillar - on a pillar

### CPI Rate Gamma

CPI Rate Gamma is the second order derivative with respect to CPI Rates. It is a symmetric matrix; where only four terms are not null.

1. Diagonal terms:
2. Cross term:

From the Gamma CPI rate chain rule, we need the following terms to be able to calculate it:

* First part of the chain rule in CPI rate Delta calculation
* Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
* New term; second order derivative of the PV to its CPI rate
* New term; second order derivative of CPI rate on the curve
* New term; first cross order derivative of CPI rate on the curve

1. Sensitivity on trade CPI rate
2. Projection on CPI rates of the Inflation curve

In between two pillars

Before first pillar/ On a pillar

The chain rule can be simplified and the CPI rate Gamma can be directly written as:

### Par Gamma

Par Gamma is the second order derivative with respect to Par rates. It is a symmetric matrix; where only four terms are not null.

.

There is a one to one relationship in between index levels and par rates; therefore derivatives to Par rates can be calculated without using the chain rule.

1. Diagonal terms:
2. Cross term:

From the Par Gamma chain rule, we need the following terms to be able to calculate it:

* First part of the chain rule in CPI rate Delta calculation
* Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
* Second order derivative of the PV to its CPI rate
* Second order derivative of CPI rate on the curve
* First cross order derivative of CPI rate on the curve
* Last part of the chain rule in Par Delta calculation
* New term; second order derivative of the CPI rate with respect to the par rate

**Not needed**

In between two pillars:

On a pillar:

Before first pillar:

### CPI Rate Gamma (direct derivation)

1. Diagonal term:

In between two pillars:

Before first pillar:

On a pillar:

1. Cross term:

In between two pillars:

Before first pillar/ On a pillar

### Par Gamma (direct derivation)

1. Diagonal term:

In between two pillars

Before first pillar:

On a pillar:

1. Cross term:

In between two pillars

Before first pillar/ On a pillar

### Cross Gamma: Par Inflation / ZC IRS

Cross Gamma is a matrix that mixes IRS and INF Deltas. In case of piecewise interpolation, the matrix has only four non null terms at maximum.

As defined in the first part, there are therefore 6 different cases to calculate Cross Gamma and project it on both curves.

In between two IRS and INF tenors:

The general formulas when an index level falls in between two curve pillars are the following:

Where:

Date of right tenor

Date of left tenor

Swap payment date

Before first tenor of Inflation curve and on an IRS tenor:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

Before first tenor of Inflation curve and in between two IRS tenors:

On a tenor of Inflation curve and IRS curve:

When an index level falls on a pillar date, the trade is only sensitive to that unique pillar and the delta formula is much simpler.

Where:

Time to maturity from payment date to pricing date using ACT/365 basis convention

Time to maturity from swap end date using 30/360 by month basis convention

On a tenor of Inflation curve and in between IRS curve tenors:

In between two Inflation tenors and on IRS tenor:

## Linear Interpolation

Where

: Reference CPI of the deal, calculated from swap start date and using the deal lag

=: Coefficient of the linear interpolation

: CPI of the current month where the index level fixes

: CPI of the month following

: Current month of the market data date

: Index lag in month

: Day of the Swap End date

: Number of days in the Swap End date month

: Swap End date

Fixed rate

Nominal

Time to maturity for the swap following using 30/360 by month basis convention

Discount factor from the payment date to the pricing date using ACT/365 basis convention

Pay or receive flag related to the Inflation leg:

### CPI Delta

CPI Delta for linear indices is the composition of two derivatives: the derivative of the PV against the two CPIs used in the linear interpolation and the projection on the CPIs of the curve.

1. Sensitivity on Index levels
2. Projection on Index levels of the curve

In between two pillars

Before first pillars

On a pillar

### CPI Rate Delta

CPI Rate Delta for linear indices is the composition of three derivatives: the derivative of the PV against the two CPIs used in the linear interpolation, the derivative of a CPI against its CPI rate and of the CPI rate sensitivity on the CPI rates of the curve. A deal is only sensitive to a maximum of two CPI rates of the curve.

1. CPI against its CPI rate
2. CPI rate against curve pillars

In between two pillars

Before first pillar- on a pillar

### Par Delta

Par Delta for linear indices is more complex because of the projection on different dates and link between index levels and par rates. A way to calculate delta is to use the chain rule: first derive regarding the two CPIs involved in the linear interpolation, then derive those against the zero coupon rate, then project the zero coupon rate sensibilities on adjacent pillars of the curve and finally derive the results against the market quotes.

Where

CPI of the trade used for pricing and interpolated from and

: CPI of the current month of

: CPI of month following

: CPI rate of the current month of

: CPI rate of month following

CPI rate of left pillar of the current bucket

Market quote of left pillar of the current bucket (par rate)

CPI rate of right pillar of the current bucket

Market quote of right pillar of the current bucket (par rate)

All the terms involved in the Delta calculation are known except . This term is a part of the Jacobian matrix which helps convert Zero sensitivity into Par sensitivity.

Jacobian Matrix construction steps:

* Derive Par rates with respect to all CPI rates used in the curve stripping (matrix)
* Derive CPI rates used in the curve stripping against CPI rates at which CPIs are expressed (matrix)
* Sum both matrices (matrix)
* Inverse matrix to get to matrix

1. Par rates function of CPI rates

Where

: Par rate for pillar n

CPI rate for pillar n

Seasonality of last published index month

Seasonality for pillar n month

Time to maturity in between the fixing date n and the reference index level date using 30/360 by month basis convention

Time to maturity in between the fixing date n and the last published index date using 30/360 by month basis convention

Coefficient of the linear interpolation

1. Pre-compute matrix

In between two pillars

Left pillar is fixed

1. Compute matrix

Based on the formula that links CPI rates used in the curve stripping; the following matrix can be built:

In between two pillars

First pillar

1. Compute matrix

After calculating matrix and matrix, we can deduce matrix by composing both matrices. This can be done using the following formula:

1. Jacobian Matrix

The final matrix to derive a CPI rate regarding Par rates and project the sensitivity on the pillar dates can be calculated by inverting .

### CPI Gamma

CPI Gamma can be calculated by applying the chain rule to the PV formula:

Terms of the CPI Gamma matrix follows exactly the same formulas as for the piecewise interpolation case except it’s applied twice; once for and once for . Contrary to Piecewise CPI Gamma, Linear CPI Gamma can’t be null; only one of the term can be null at a time ( or ).

1. Diagonal terms:
2. Cross term:

### CPI Rate Gamma

CPI Rate Gamma can be calculated by applying the chain rule to the PV formula:

Terms of the CPI Rate Gamma matrix follows exactly the same formulas as for the piecewise interpolation case except it’s applied twice; once for and once for .

1. Diagonal terms:
2. Cross term:

### Par Gamma

Par Gamma for linear indices is as complex as Par Delta because of the index level dependency to previous Par rates when stripping the Inflation curve. A way to calculate Gamma is to use the chain rule up to the CPI rate Gamma and then compose it with the Hessian matrix to translate CPI rate Gamma into Par Gamma.

Starting from the PV formula, we can deduce that in the case of a Linear interpolation:

And

From the Par Gamma chain rule above, we need the following terms to be able to calculate it:

* First part of the chain rule in CPI rate Delta calculation
* Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
* Second order derivative of the PV to its CPI rate
* Second order derivative of CPI rate on the curve
* First cross order derivative of CPI rate on the curve
* Last part of the chain rule in Par Delta calculation
* New term; second order derivative of the CPI rate with respect to the par rate
* New term; first order cross derivative of the CPI rate with respect to the par rates

The two new terms can’t be calculated directly but the inverse derivatives can.

1. Express matrix

In between two pillars

Where:

=

First pillar

Where:

1. Hessian Matrix

After building multiple matrices which represent the second derivative of a Par rate with regards to CPI rates, we can deduce the Hessian matrix.

**matrix:**

|  |  |  |
| --- | --- | --- |
| Second derivative of Par rate 1 to CPI rates | Second derivative of Par rate 2 to CPI rates | Second derivative of Par rate 3 to CPI rates |
|  |  |  |

Hessian Matrix:

|  |  |  |
| --- | --- | --- |
| Second derivative of CPI rate 1 to Par rates | Second derivative of CPI rate 2 to Par rates | Second derivative of CPI rate 3 to Par rates |
|  |  |  |